TEMPERATURE COMPENSATION OF THE COMPOUND PENDULUM

Alan Emmerson

The period of the free swinging compound pendulum is :

$$P = 2\pi\sqrt{\frac{I}{hMg}}\left(1+\frac{1}{4}\sin^2\frac{\alpha}{2}+\frac{9}{64}\sin^4\frac{\alpha}{2}+\cdots\right) \qquad 1.$$

where: I is the mass moment inertia of the pendulum about the pivot axis
Mg is the restoring force applied to the displaced pendulum bob, principally determined by M the mass of the pendulum,
α is the angular amplitude of swing and
h is the distance from the pivot axis to the centre of mass of the pendulum.
Mg is conventionally considered as the "apparent weight"

For convenience let us rewrite this as:

$$P = \sqrt{\frac{I}{h}}\left(1+f(\alpha)\right)\frac{2\pi}{\sqrt{Mg}}$$ and rewrite this again as: $P = \sqrt{\frac{I}{h}}\beta$

The rate of change of period with temperature will be given by:

$$\frac{dP}{dT} = \beta\frac{d\sqrt{\frac{I}{h}}}{dT} + \sqrt{\frac{I}{h}}\frac{d\beta}{dT} \qquad 2.$$

On the right hand side of this equation, the left hand term contains the primary causes of the variation of period with temperature. These are the dimensional changes due to temperature.

The right hand term contains all the messy second order effects that contribute to the change of amplitude of swing with temperature.

It is quite implausible that these terms should have the same magnitude and opposite signs.

Accordingly, if $\frac{dP}{dT}$ is to be zero, then these terms must both be zero

$\sqrt{\frac{I}{h}}$ is non zero so that for the right hand term to be zero $\frac{d\beta}{dT}$ must be zero. That is:

$$\frac{d}{dT}\left(\left(1+f(\alpha)\right)\frac{2\pi}{\sqrt{Mg}}\right) = 0$$

This requires that $\frac{dMg}{dT}$ and $\frac{d\alpha}{dT}$ must both be zero. That is, the apparent weight must not change with temperature and neither must the amplitude of swing.

The theoretical compound pendulum, the pendulum to which the equation 1 applies, swings in a vacuum. There is no buoyancy. $\frac{dMg}{dT}$ is zero. In a real pendulum that will not be so.

But even in the theoretical compound pendulum $\frac{d\alpha}{dT}$ is not zero. If the dimensions of the pendulum change, but the energy of the pendulum remains constant, the amplitude of swing must change.

Now consider the dimensional changes and the term $\beta\frac{d\sqrt{\frac{I}{h}}}{dT}$ in equation 2

For a pendulum consisting of a rod and cylindrical bob of lengths ***l_1 , l_2*** radii ***r_1 ,r_2*** and masses ***m_1 ,m_2*** respectively, suspended a distance ***e*** above the top of the rod

$$I = m_1(\frac{1}{3}l_1^2 + \frac{1}{4}r_1^2 + el_1 + e^2) + m_2(\frac{1}{3}l_2^2 + l_1l_2 + l_1^2 + \frac{1}{4}r_2^2 + e^2 + 2el_1 + el_2) \text{ and}$$

$$h = e + (\frac{1}{2}l_1m_1 + l_1m_2 + \frac{1}{2}l_2m_2) \Big/ (m_1 + m_2)$$

Unless my algebra has let me down. So that:

$$\frac{I}{h} = [m_1(\frac{1}{3}l_1^2 + \frac{1}{4}r_1^2 + el_1 + e^2) + m_2(\frac{1}{3}l_2^2 + l_1l_2 + l_1^2 + \frac{1}{4}r_2^2 + e^2 + 2el_1 + el_2)] \bullet \frac{(m_1 + m_2)}{(\frac{1}{2}l_1m_1 + l_1m_2 + \frac{1}{2}l_2m_2) + e(m_1 + m_2)}$$

The lengths ***l_1, l_2*** the radii ***r_1 ,r_2,*** and the distance ***e*** all change with temperature. I am not even prepared to write out $\beta\frac{d\sqrt{\frac{I}{h}}}{dT}$

Does anyone really think they can fully compensate a pendulum for changes in temperature?