BUOYANCY

THE LEGEND OF ARCHIMEDES AND EUREKA
CIRCA 250BC

Once upon a time, in the city state of Syracuse on the island of Sicily, King Hiero II ordered a new golden crown. The king became suspicious that the goldsmith who was making the crown might cheat him by making it out of a mixture of gold and a cheaper metal rather than out of pure gold.

Being a king of infinite wisdom he consulted the most famous engineer and mathematician of the time - one Archimedes of Syracuse. When the crown was delivered, how would he be able to tell if it was made of an alloy of gold or out of pure gold?

Archimedes saw the key to the problem straight away. Gold was the most dense metal known in Syracuse. A mixture of gold with any other metal would be less dense. That is a given volume of the mixture would weigh less than the same volume of pure gold. So all Archimedes had to do was measure the density of a sample of pure gold then measure the density of the crown.

But, how to measure density? They could weigh the sample of gold and they could weigh the crown . They might be able calculate the volume of the gold sample if it were say a cube, but they could not calculate the volume of anything so irregular as a crown. Archimedes worried about this for weeks. Time was running short, the crown would soon be delivered.

One evening, as Archimedes lowered himself into his bath, which had been rather over filled by his servant. The water overflowed. Suddenly Archimedes had his answer. Put the crown in the bath, measure the volume of water displaced by the crown and you would know the volume of the crown. Archimedes ran naked out into the street shouting "Eureka eureka!" ((I have found it!).

Of course the matter did not end there. How would he measure or calculate the volume of water that overflowed. Some of it always seemed to spill. Archimedes made an apparatus like a beam balance for carefully lowering objects into containers filled with water. In using this apparatus he noticed that the weight of any object seemed to reduce when it was lowered into the water. Archimedes sat down and thought very hard about this . He deduced that the loss of weight would be exactly equal to the weight of the liquid displaced by the immersed object. Now he had his answer. He would weigh the sample of gold in air then in water and this would tell him its density as a multiple of the density of water. Then he could do the same for the crown.

Now this legend really does need some adjustment. In the first place, all the good king Hiero had to do was personally watch the goldsmith melt and pour the gold. Secondly, what self respecting engineer would run out into the dusty street with wet feet.

Nevertheless, we do, 2250years later, still use **Archimedes' Principle**.

When an object is partly or wholly immersed in a fluid:

- *the volume of fluid displaced is equal to the volume of the immersed portion of the object*
- *the object experiences an up-thrust equal to the weight of the fluid displaced.*

The whole phenomena of an object apparently losing weight when immersed in a fluid is called "buoyancy".

We call the density of a substance divided by the density of water the "specific gravity" of the substance.

$$SG_{substance} = Density_{substance} / Density_{water}$$

The density of water is 1000 kg /cubic metre.

The specific gravity of gold is 19.3 Only rhenium is more dense, having a SG of 21.0

Archimedes' Method

We must first assume that the up-thrust on the crown when immersed only in air is negligible. This is reasonable because the density of air is very low compared to the density of gold. Then if:

W_{ca} = the weight of the crown in air

W_{cw} = the apparent weight of the crown when immersed in water

V = the volume of the crown

ρ_c = the average density of the crown

ρ_w = the density of water, and

g = the acceleration due to gravity.

$$W_{ca} = \rho_c Vg$$

The weight of the water displaced by the crown = $\rho_w Vg$

$$W_{cw} = \rho_c Vg - \rho_w Vg$$

$$\frac{W_{cw}}{W_{ca}} = \frac{\rho_c Vg - \rho_w Vg}{\rho_c Vg}$$

$$\frac{W_{cw}}{W_{ca}} = \frac{\rho_c - \rho_w}{\rho_c}$$

$$\frac{W_{cw}}{W_{ca}} = 1 - \frac{\rho_w}{\rho_c}$$

$$\frac{\rho_w}{\rho_c} = 1 - \frac{W_{cw}}{W_{ca}}$$

$$\frac{\rho_w}{\rho_c} = \frac{W_{ca} - W_{cw}}{W_{ca}}$$

$$\frac{\rho_c}{\rho_w} = \frac{W_{ca}}{W_{ca} - W_{cw}}$$

So, if the crown weighed 1kg in air and appeared to weigh only 947.5grams in water the SG of the crown was 19.03 and was almost certainly pure gold. Notice how sensitive this method is to inaccuracies in weighing. If the weight of crown in water was measured to be 937.5 grams the SG of the crown would have worked out at 16 and the goldsmith would have lost his head.

Finally we should note that Archimedes could not possibly have used the algebraic method just outlined. The first equation requires the laws of Newton who was 1900 years into the future.