IDENTIFICATION AND EVALUATION OF $\prod$
BY THE ANCIENTS

The earliest Indo European civilizations were aware that the area of a circle is proportional to the square of its radius and that the circumference is proportional to the diameter. It is not clear when they first realised that the constants of proportionality were the same. (The symbol $\prod$ is comparatively recent being introduced by Jones in 1706 and popularized by Euler in C18th)

The method of approaching $\prod$ by considering the perimeters of inscribed and circumscribed polygons as upper and lower bounds of the circumference of the circle was used by Antiphon and his contemporary Bryson of Heraclea. They would have understood $\prod$ to have a value of around 3.2

Later, Archimedes of Syracuse rigorously established the equivalence of the two ratios. He also improved on the procedure of Bryson and Antiphon when he showed the effect of doubling the number of sides on the polygons. He thereby established a procedure which when repeated often enough would in principle give $\prod$ to as many digits as required. Archimedes started with hexagons, yielding the determination $3<\prod<2\sqrt{3}$. By doubling the number of sides four times he obtained the estimate $\prod= 3.14$. There is some evidence that Archimedes tried starting with a decagon and got $\prod = 3.1416$.

Archimedes' method is very slow as it involves extracting roots and converges slowly, but, it was the basis of all estimates up until mid C17th .

In C16th, van Ceulen obtained a 32 digit estimate using a polygon with 2^{62} sides His value for $\prod$ is called the Ludolphian number.

The 1986 estimate has 134,217,000 digits

Fitters and turners use the ratio 355/113. This gives a value accurate to 1 part in 4 million.

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