THE ENERGY PRINCIPLE

How many times have you seen a passage in dynamics reading

"conservation of energy gives $\frac{1}{2}mv^2 = mgh$" as if this were some fundamental principle or axiom.?

To say this was wrong would be something of an overstatement, but at best it is a piece of shorthand which brushes over and conceals some important physics.

Energy is defined as the ability to do work. When work is done on a system of particles or a body, the energy of the body is said to increase by the amount of the work done.

Work is done by a force on a body when that force moves with the boundary of the body. The amount of work done by a force F moving a distance ds with the boundary of a body, is defined as

$W = \int_{s_0}^{s_1} F ds$ and accordingly the increase in the energy of the body is

$$\Delta E = \int_{s_0}^{s_1} F ds$$

If, and only if, the body accelerates freely under the action of the force, $F = \frac{dmv}{dt}$ so that

$$\Delta E = \int_{s0}^{s1} \frac{dmv}{dt} ds$$

If the mass of the body is constant $\Delta E = m\int_{s0}^{s1} \frac{dv}{dt} ds$

$$= m\int_{t0}^{t1} \frac{dv}{dt}\frac{ds}{dt} dt$$

$$= m\int_{v0}^{v1} \frac{dv}{} \frac{ds}{dt}$$

$$= m\int_{v_0}^{v_1} v dv$$

$= \frac{1}{2}m\left(v_1^{\ 2} - v_0^{\ 2}\right)$ which we know as the change in kinetic energy

Thus when a force F moves with the boundary of a body of constant mass that accelerates freely the change in kinetic energy is given by $\frac{1}{2}m\left(v_1^2 - v_0^2\right) = \int_{s_2}^{s_1} Fds$

This is the Principle of Energy. *The increase in the kinetic energy of a free body is equal to the sum of the works done on the body by each of the forces acting upon it.*

In particular, if F is a constant value mg and moves a distance h with its point of application, then

$$\frac{1}{2}m\left(v_1^2 - v_0^2\right) = \int_0^h Fds = mgh$$

which would be the case if F were the force of gravity.

There are cases other than that in which the body accelerates freely. Suppose that the body were a link with one end fixed in space. A force F is applied to the free end of the link which then stretches a distance s. The force require to accomplish this extension is not constant but varies according to Hooke's law F = ks

The increase in the energy of the body is equal to the work done by the force in creating the extension.

ie $$\Delta E = \int_{s_0}^{s_1} Fds$$

$$\Delta E = \int_0^s ksds$$

$$= \frac{1}{2}ks^2$$

One might conceive of a lever and weight arrangement such that $\frac{1}{2}ks^2 = mgh$

The Principle of Energy is not the same as the principle of the conservation of energy, which hails from a more general context, and says that energy cannot be created or destroyed and that, in consequence, any change in the mechanical energy of an isolated system must represent a conversion from or into some other form of energy (usually heat).